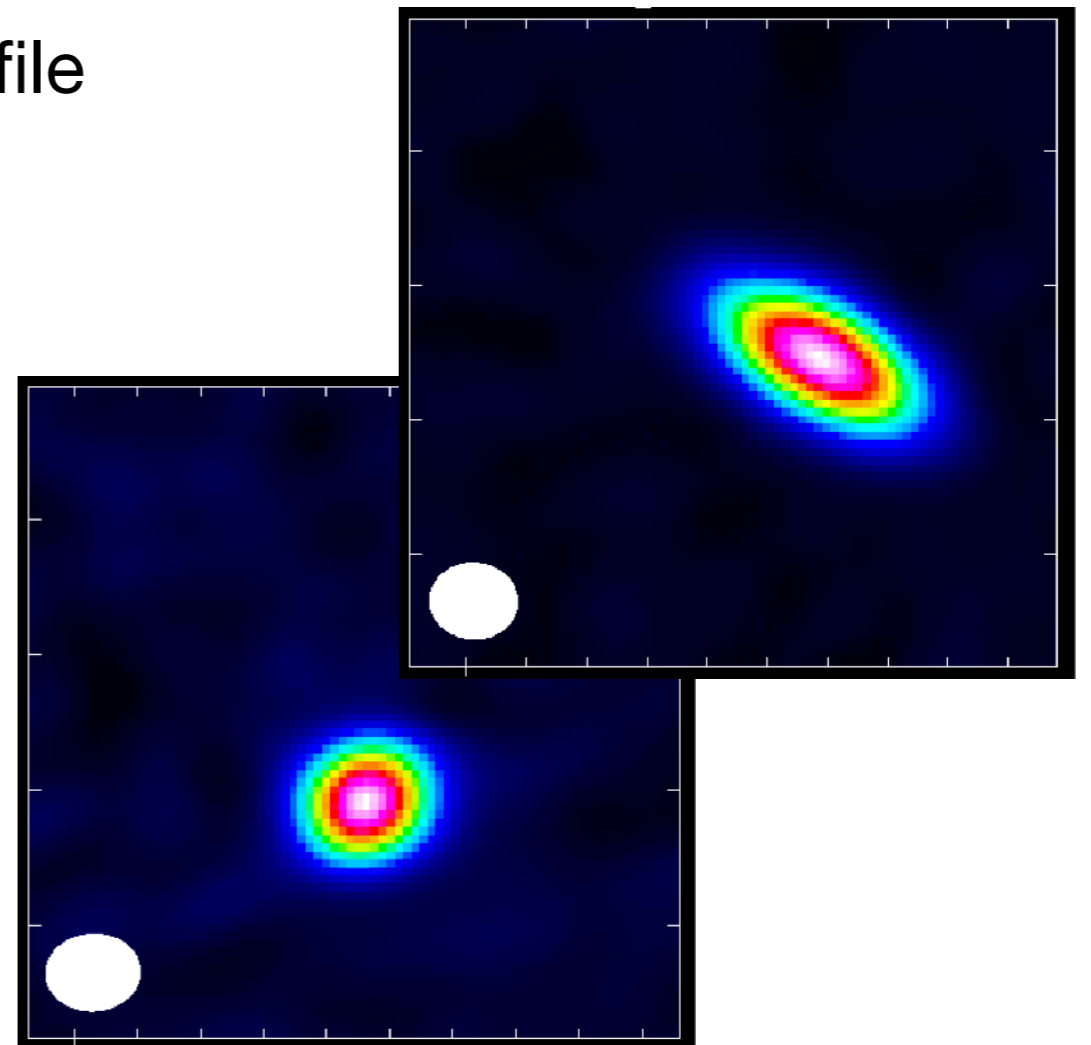


Dust

# Interpreting and modeling

For extracting flux/morphology from simple Gaussian images and point sources: CASA `imfit()` and `uvmodelfit()`

- `imfit()`: fit elliptical gaussian, output in file
- `uvmodelfit()`:
  - Define type:
    - Point source
    - Gaussian ellipse
    - Disk (sharp edge)
  - Provide initial guess (flux, xoff, yoff, bmajaxratio, bpa)
  - Provide number of iterations (check convergence)



**Dust**

# Interpreting and modeling

- Basic radiative transfer
  - Convert integrated flux to dust mass

$$M_{\text{dust}} = \frac{F_{\nu} d^2}{\kappa_{\nu} B_{\nu}(T_{\text{dust}})}$$

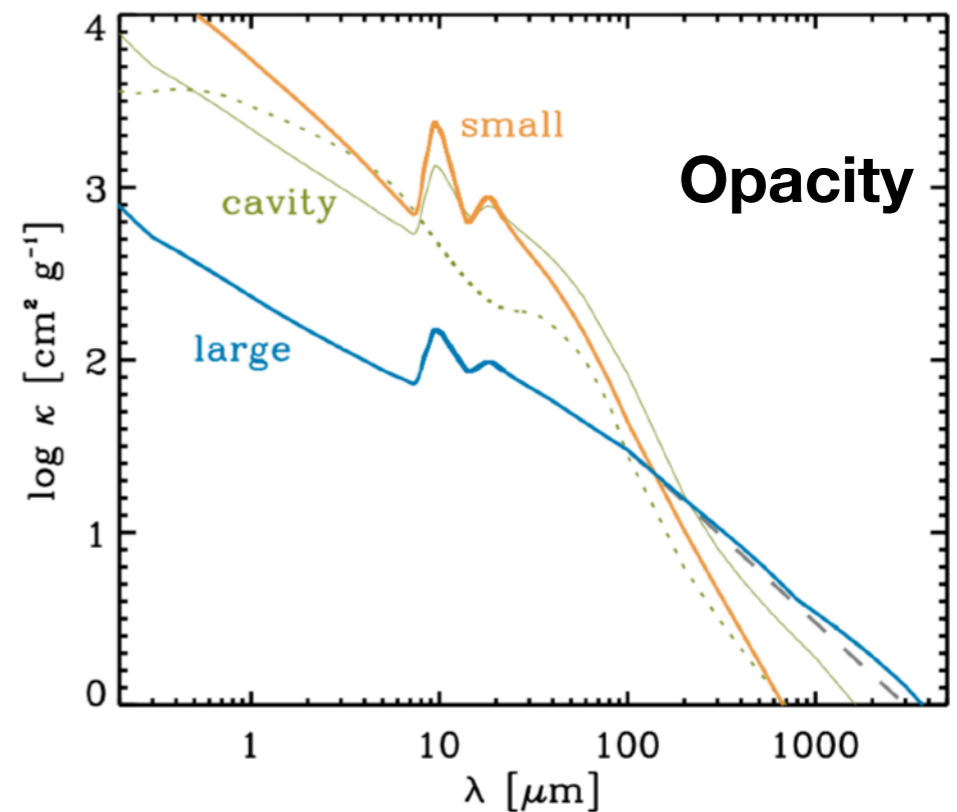
$$F_{\nu} = \int I_{\nu} d\Omega = \frac{1}{d^2} \int_{R_{\text{in}}}^{R_{\text{out}}} I_{\nu} 2\pi r dr$$

$$I_{\nu} = B_{\nu}(T(r))(1 - e^{-\tau(r)})$$

$$B_{\nu}(T(r)) = \frac{2\nu^2 k_B T(r)}{c^2}$$

$$\tau(r) = \frac{\kappa_{\nu} \Sigma_d(r)}{\cos i}$$

$$M_{\text{dust}} = \int_{R_{\text{in}}}^{R_{\text{out}}} \Sigma_d(r) 2\pi r dr$$



Hildebrand et al. 1983

Beckwith et al. 1990

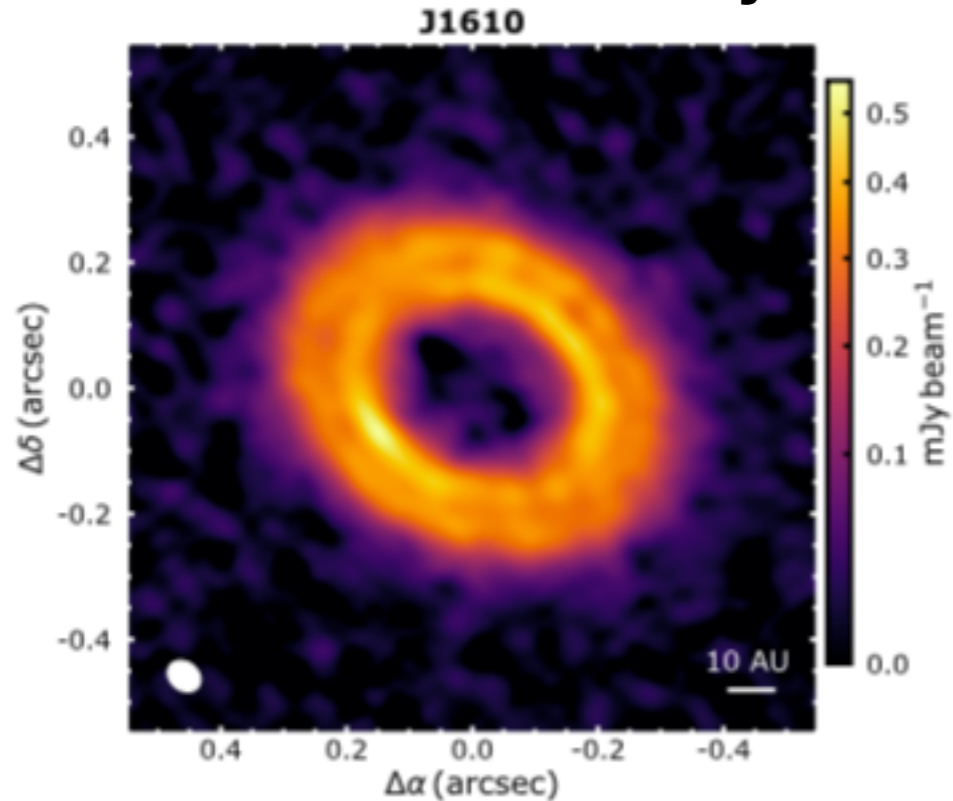
Francis & van der Marel 2020 (3.2)

**Dust**

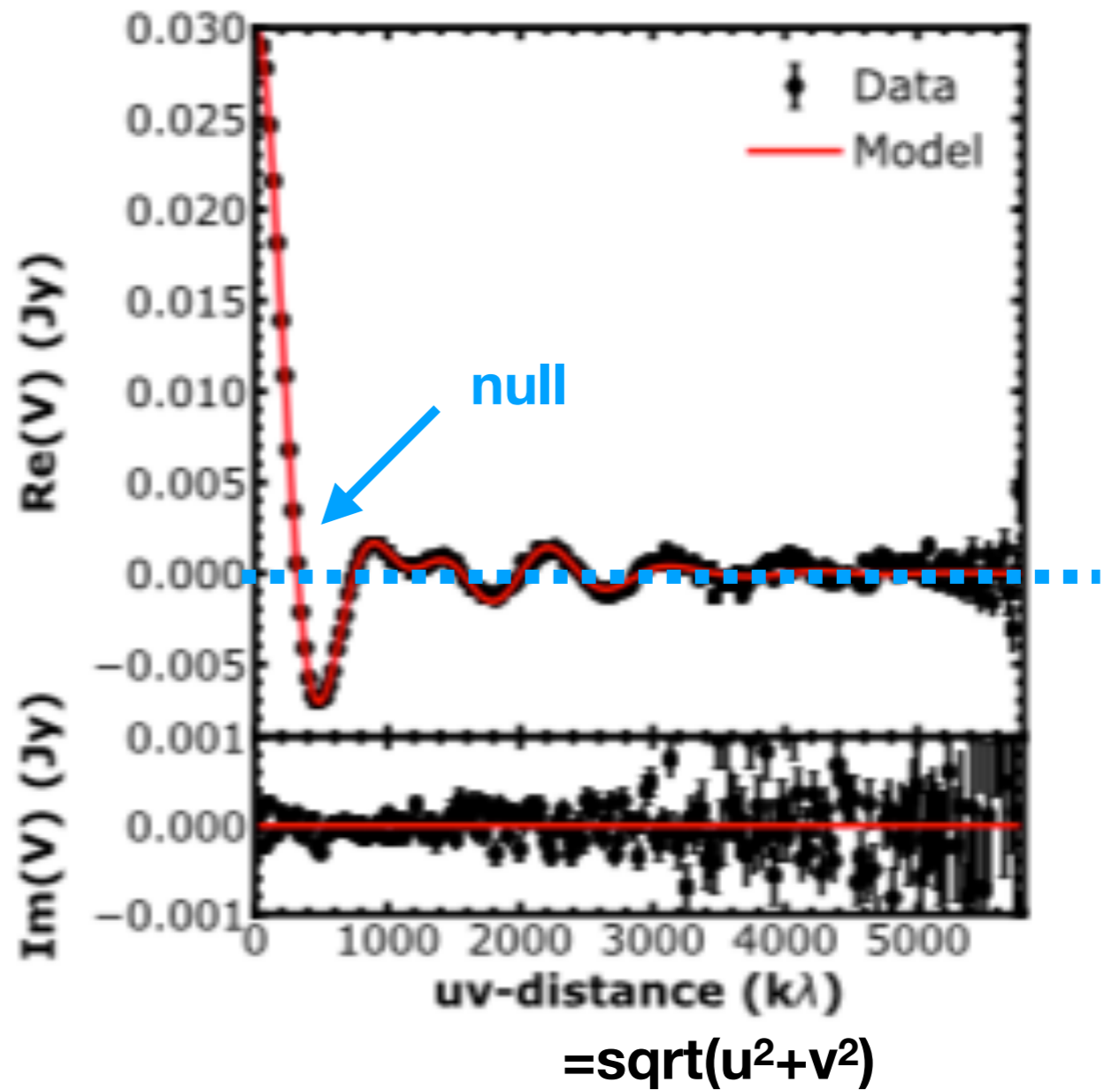
# Interpreting and modeling

For more complex structure: modelling

Dust disk with inner cavity



Null crossing is indicator of a dust ring



Imaginary is zero for axisymmetric disk

**Dust**

# Interpreting and modeling

## Intensity profile

- Intensity profile is parametrized
- Simple and fast
- Many details on structure (gap location, width, edge shape, etc.)
- Both 1D (radial) or 2D (radial+azimuthal) possible
- No knowledge of star or dust required

## Radiative transfer

- Intensity profile is computed
- Physical information: dust surface density, dust mass, grain size distribution and temperature
- 2D (radial+vertical) or 3D (radial+vertical+azimuthal) structure
- Code required
- Slow
- Combination with SED
- Uses star for radiation field

**Dust**

# Interpreting and modeling

## Image plane comparison

- Intensity model convolved with beam and subtracted from data
- Intuitive
- Fast
- May suffer from imaging artefacts and missing spatial scales
- Easy to code in python

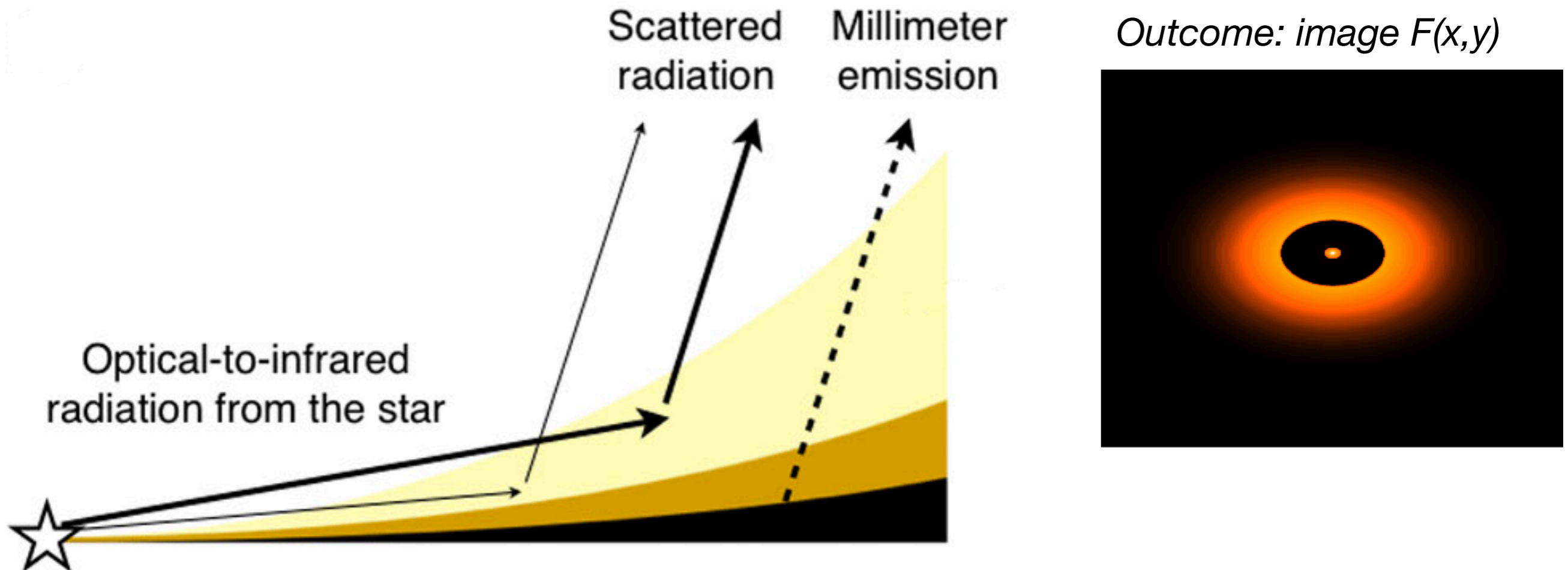
## Visibility plane comparison

- Intensity model Fourier-transformed to  $u,v$  plane and compared with visibility data
- Non-intuitive
- More detail and more accurate w.r.t. imaging artefacts and spatial scales
- Slow: usually MCMC
- Azimuthal variations very tricky to identify and model

Dust

# Interpreting and modeling

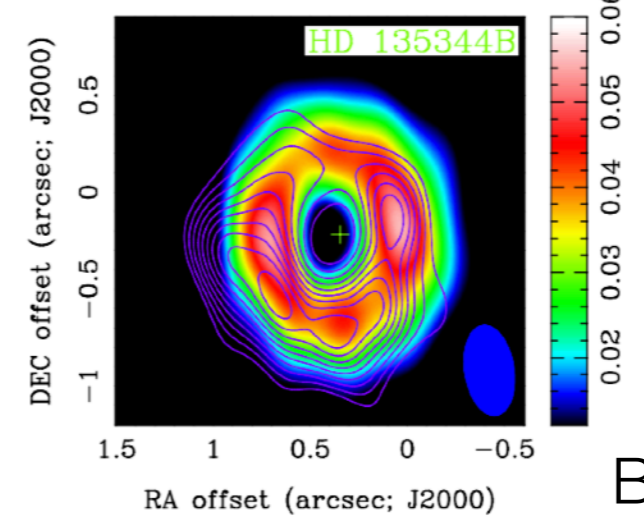
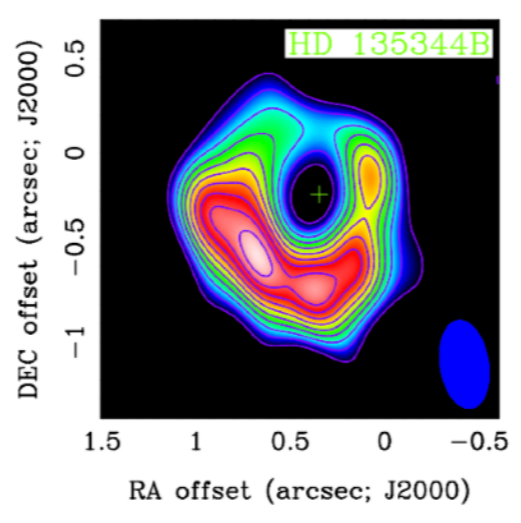
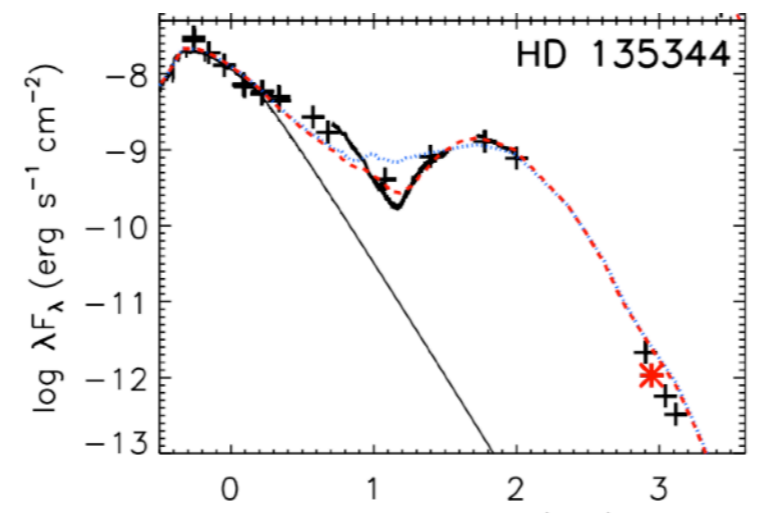
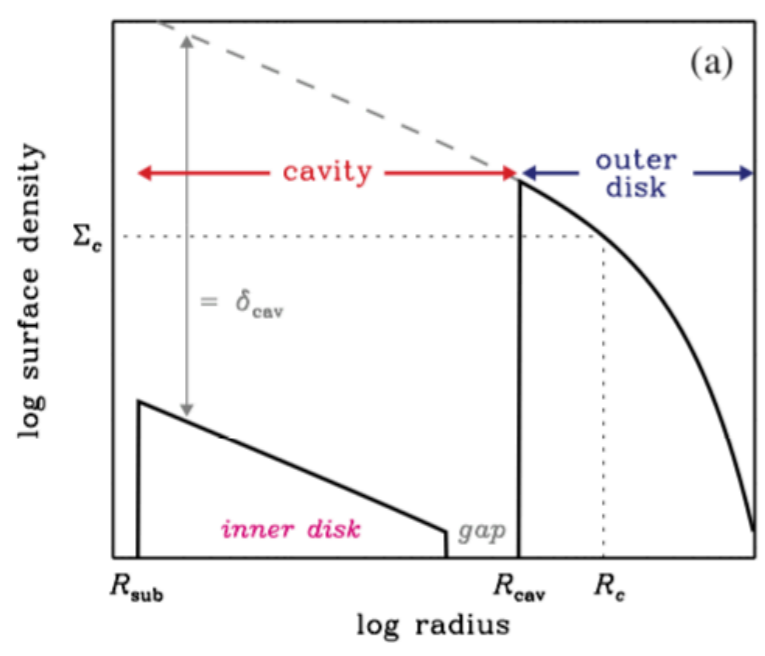
- Full radiative transfer: use 2D/3D structure disk to compute optical depth and temperature in each location and set dust opacity, to compute expected  $F(r, \theta)$  ( $F(x, y)$ )



Dust

# Interpretation and modeling

- Full radiative transfer: example codes are RADMC-3D, MCFOST, DIAD, HOCHUNK3D....
- Fitting: dust surface density and vertical profile
- Comparison: 2D intensity model + SED



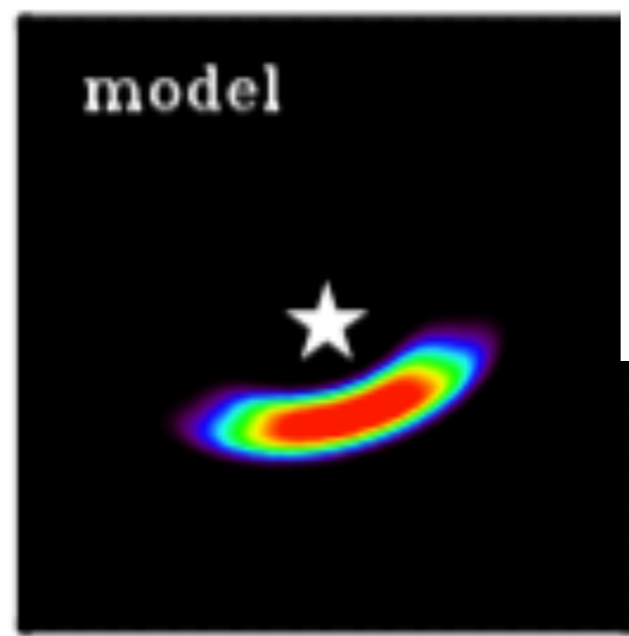
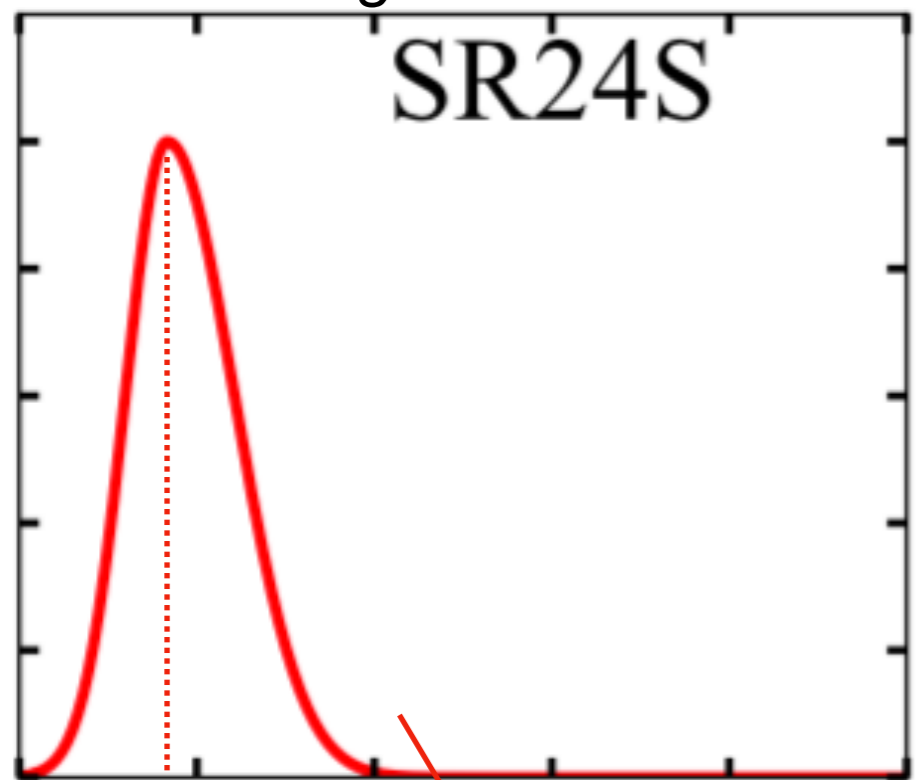
Dust

# Interpretation and modeling

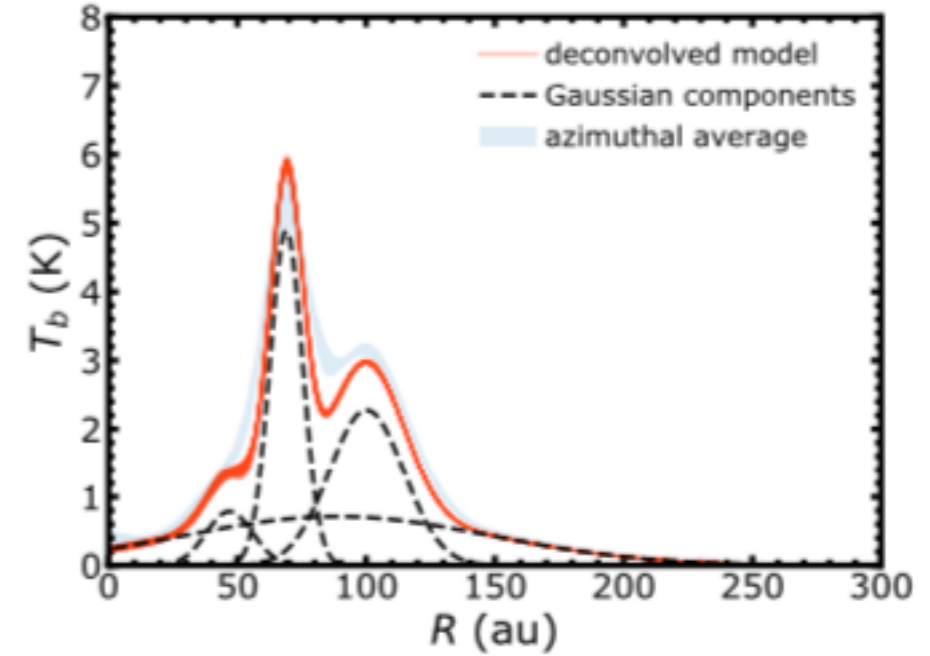
- Intensity profile: parametrization examples

*Multiple rings: multiple gaussians*

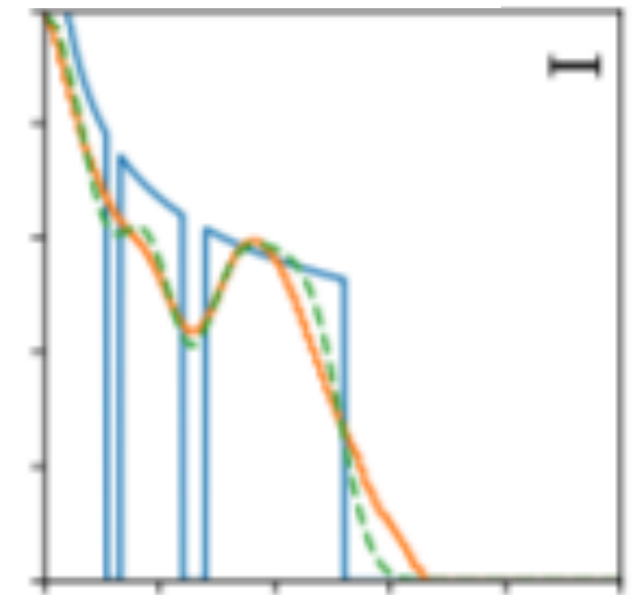
*2 radial half-gaussians*



*2D asymmetry:  
4th power gaussians in r, phi*



*Multiple rings:  
dips in power-law*



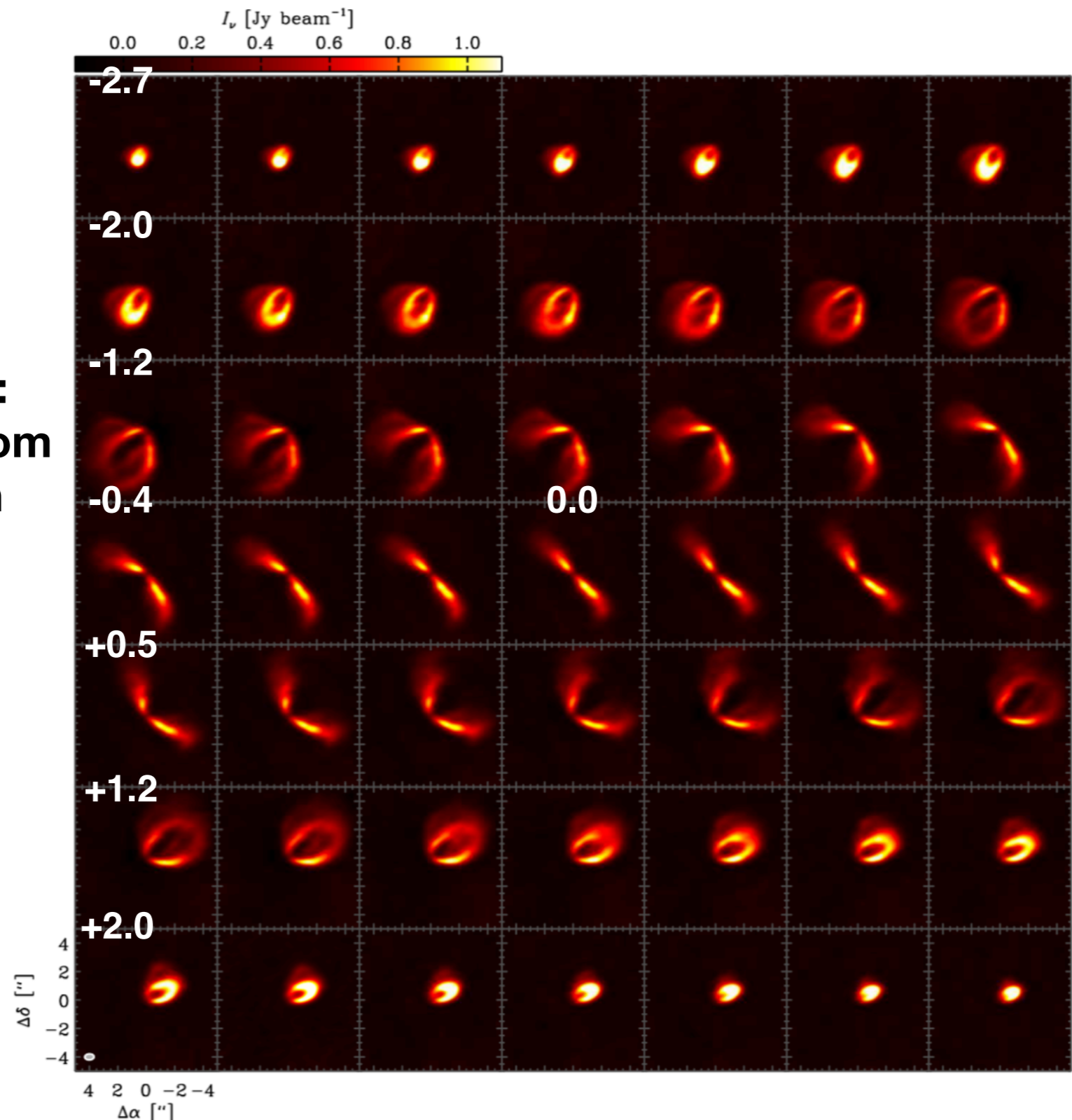
Parametrization:

$$I(r) = \begin{cases} C \exp\left(-\frac{(r-r_{\text{peak}})^2}{2\sigma_{\text{int}}^2}\right) & \text{for } r \leq r_{\text{peak}} \\ C \exp\left(-\frac{(r-r_{\text{peak}})^2}{2\sigma_{\text{ext}}^2}\right) & \text{for } r > r_{\text{peak}} \end{cases}$$

**Gas**

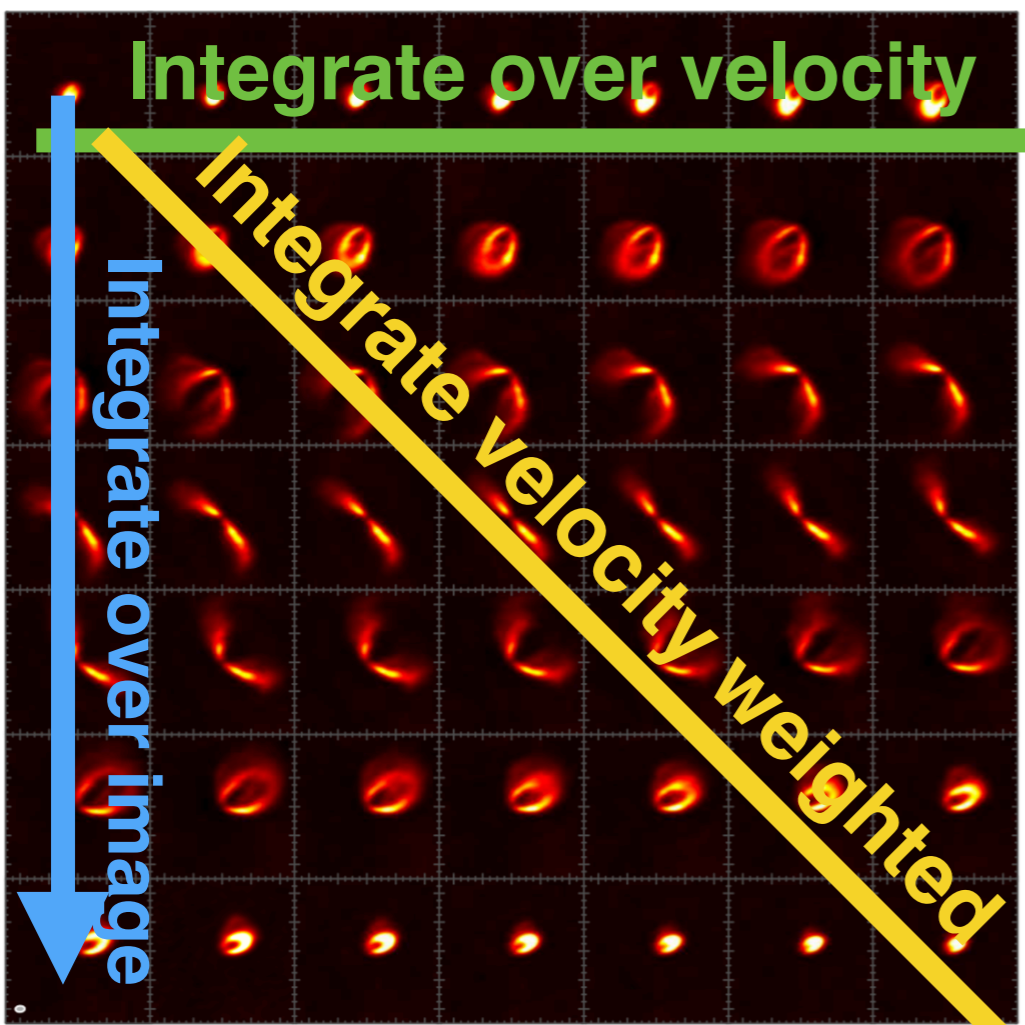
# Interpreting and modeling

**$^{12}\text{CO}$  image cube:  
Butterfly pattern from  
Keplerian motion**

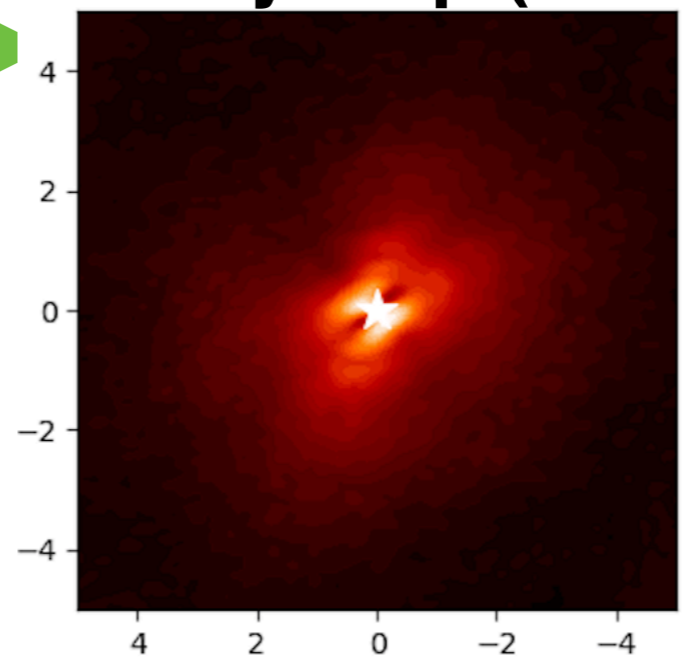


**Gas**

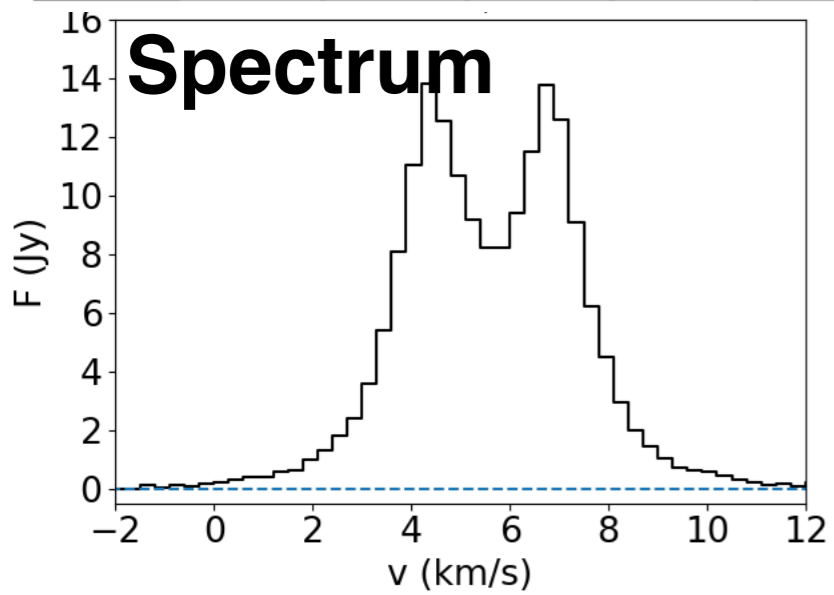
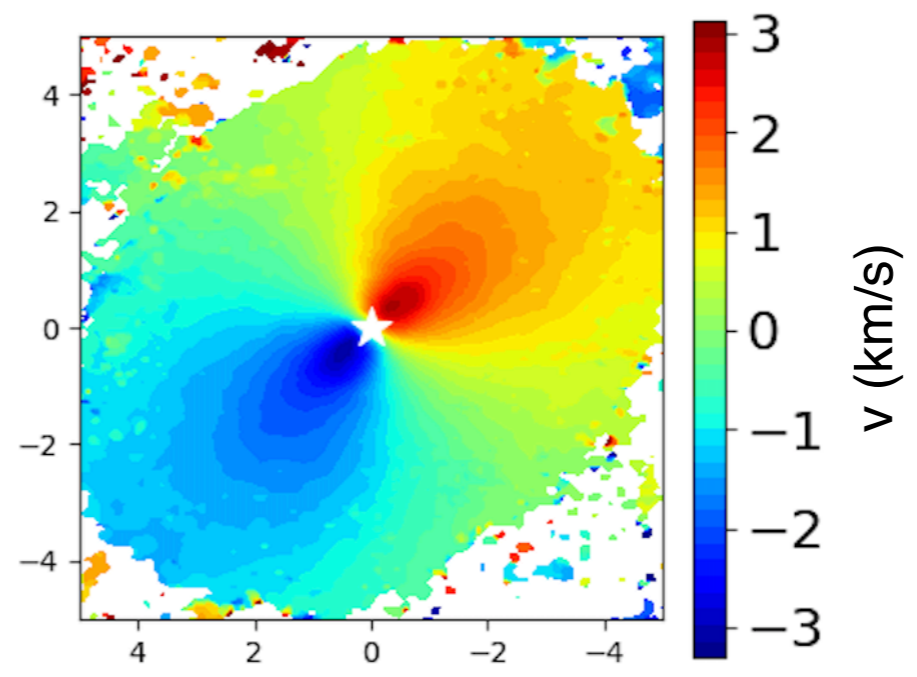
# Interpreting and modeling



Intensity map (moment 0)



Velocity map (moment 1)

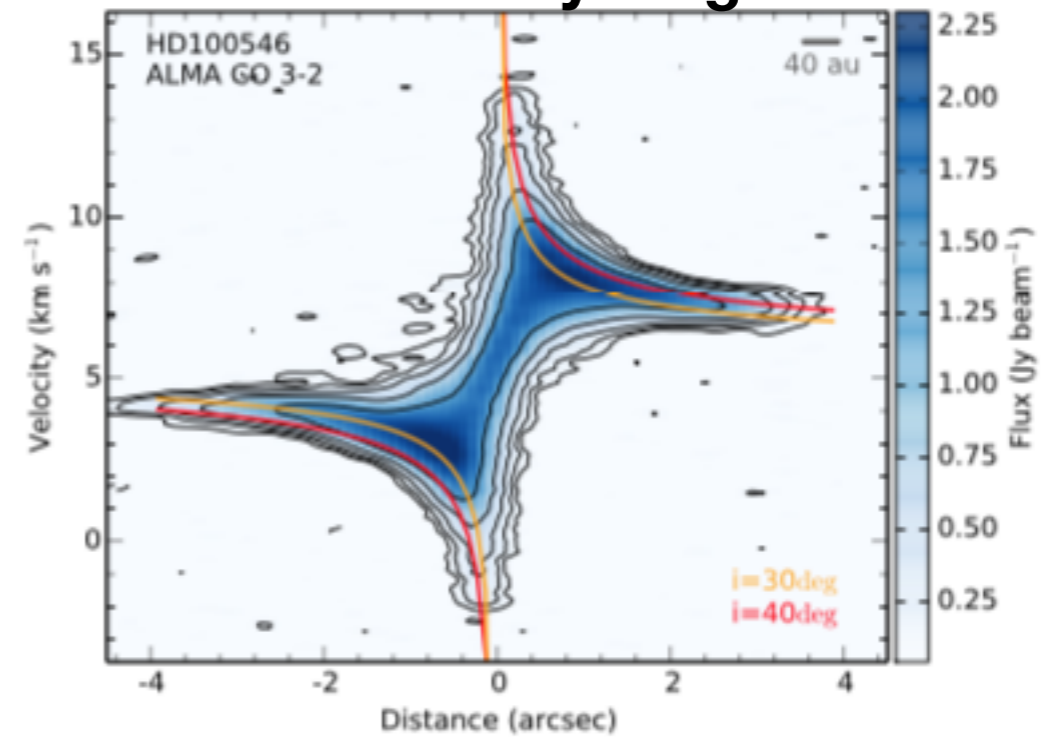


**Gas**

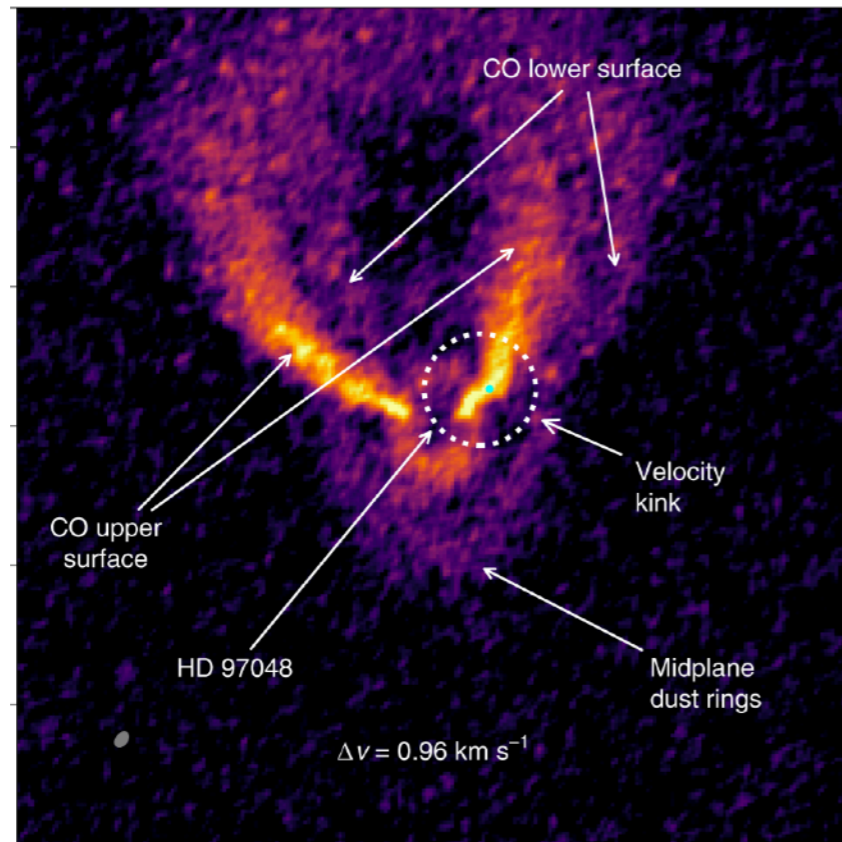
# Interpreting and modeling

- Disk orientation: inclination, position angle
- Kinematics: dynamical mass, warps, deviations

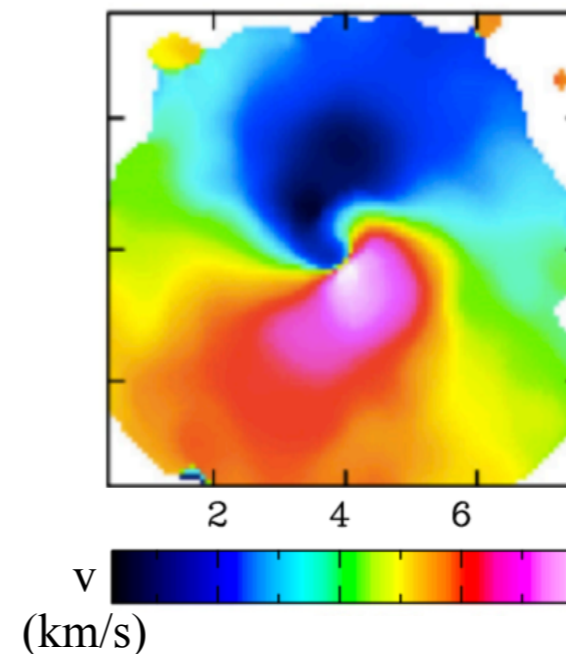
**Position-velocity diagram**



**Velocity kink in channel**



**Moment-1**



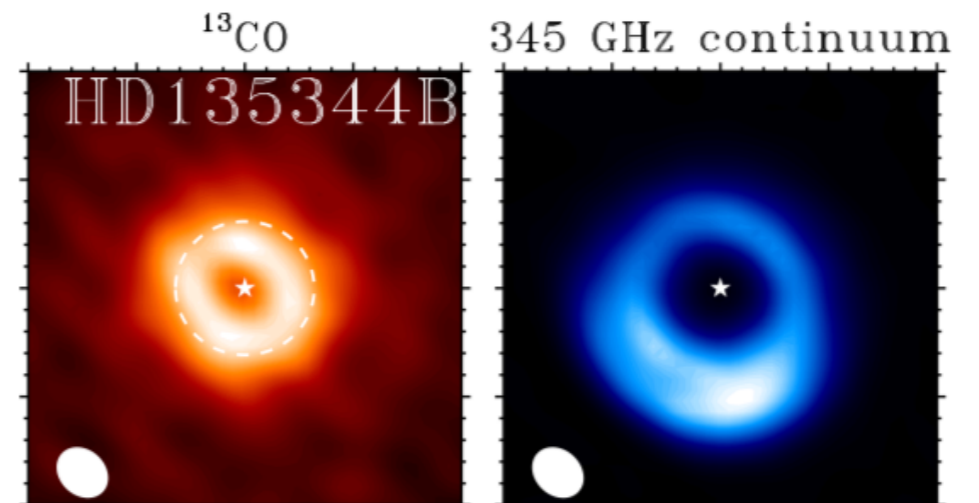
Casassus et al. 2013  
Pineda et al. 2014  
Pinte et al. 2018

Gas

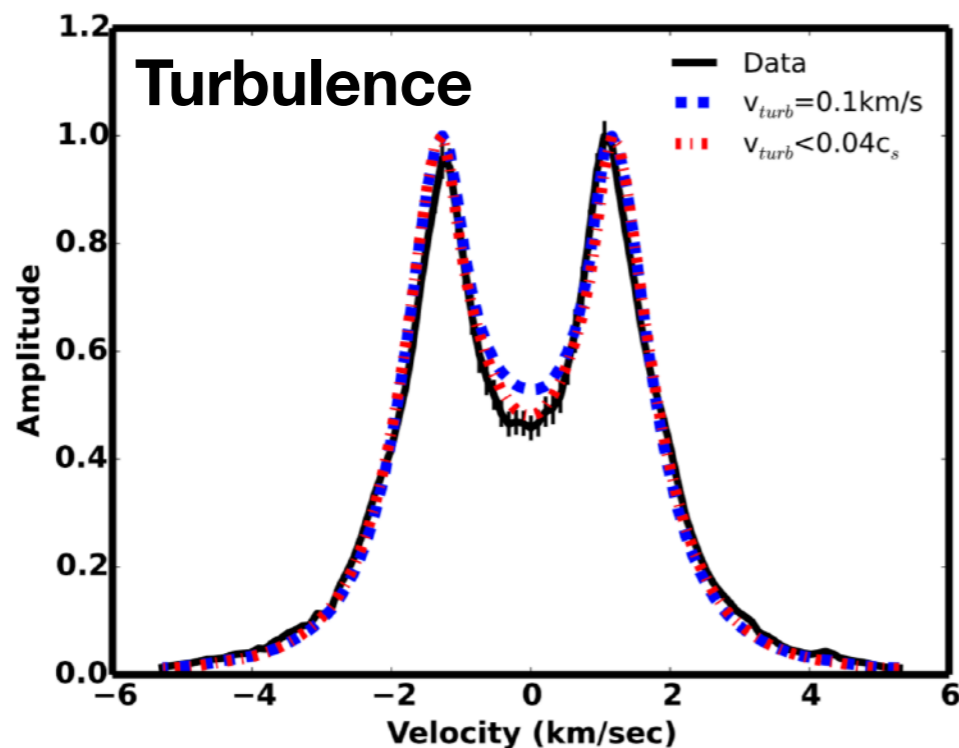
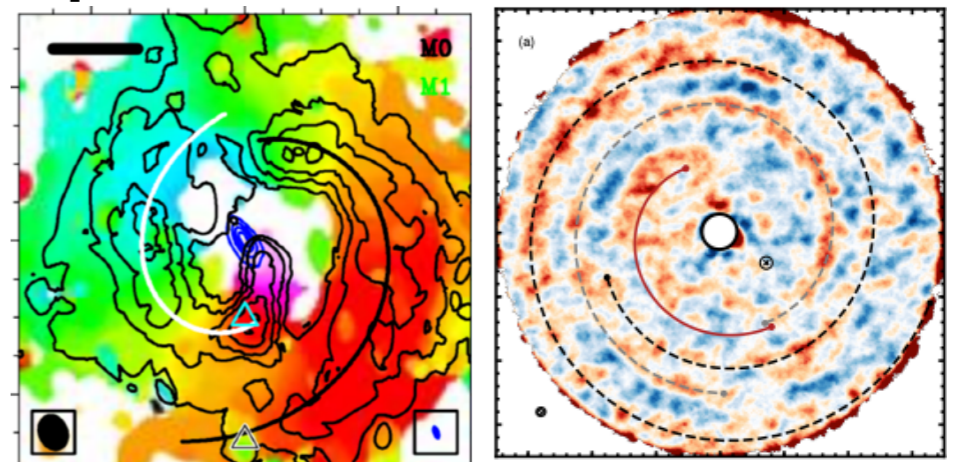
# Interpreting and modeling

- Spatial distribution gas vs dust: gaps, dust traps
- Surface: spiral arms
- Turbulence: line broadening
- Chemistry: different molecules

## Gas vs dust



## Spirals

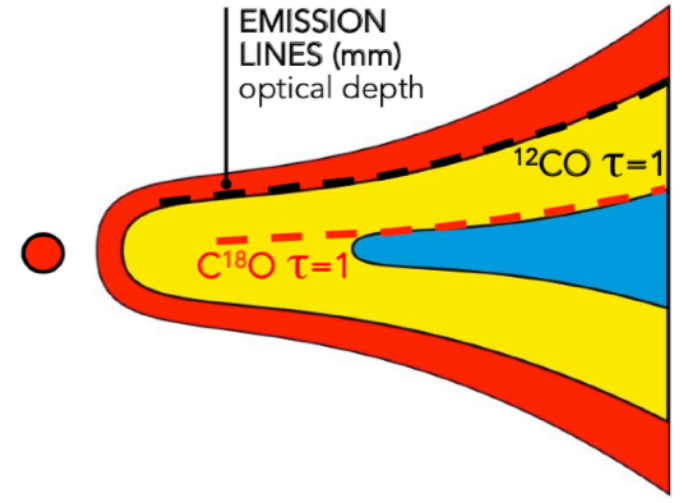
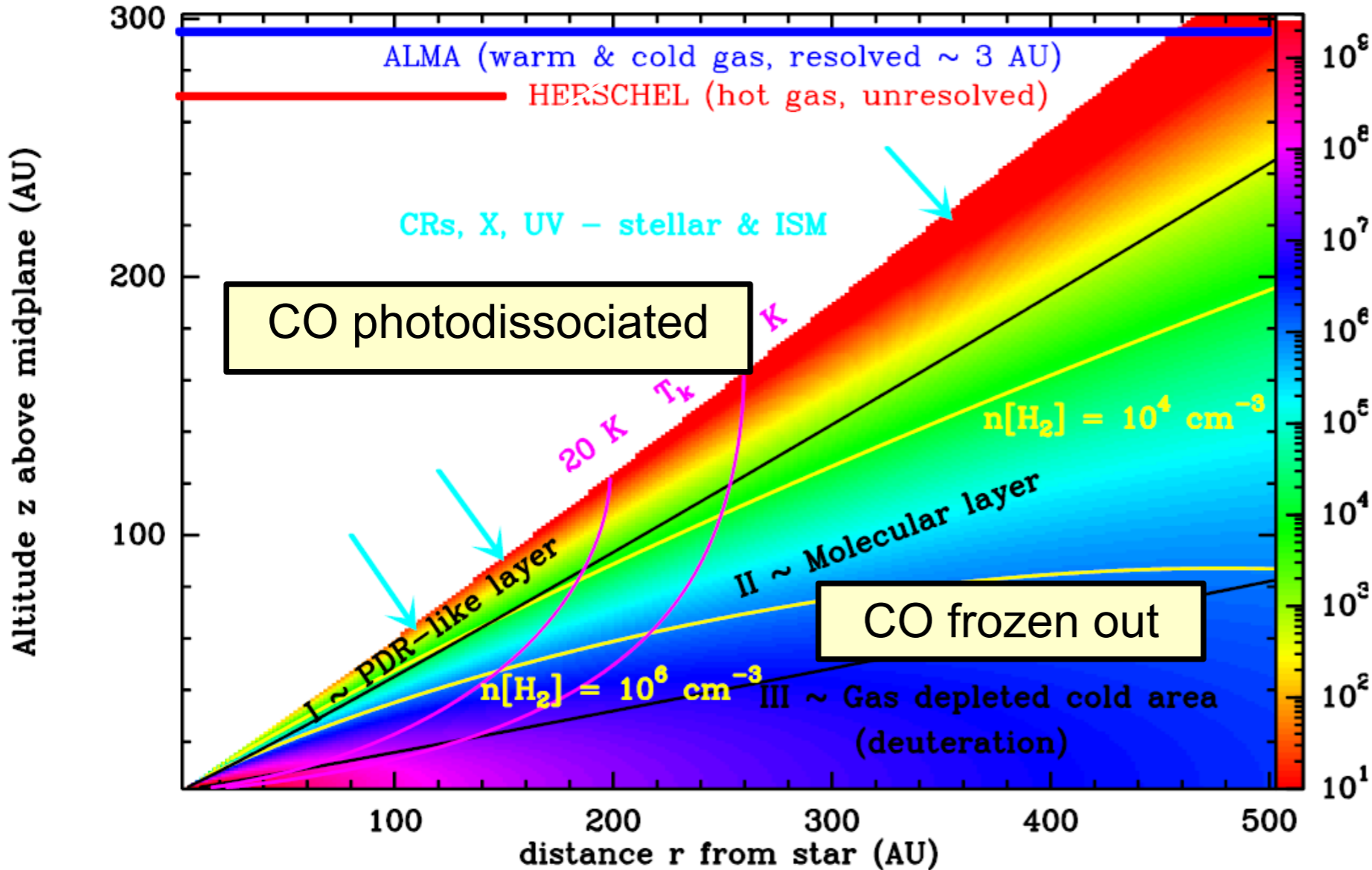


Van der Marel et al. 2016  
Tang et al. 2017  
Teague et al. 2019  
Flaherty et al. 2015

Gas

# Interpreting and modeling

- Gas surface density: CO isotopologues as H<sub>2</sub> tracer



=> need physical-chemical model to quantify  $\Sigma_{\text{gas}}(r)$  from CO

**Gas**

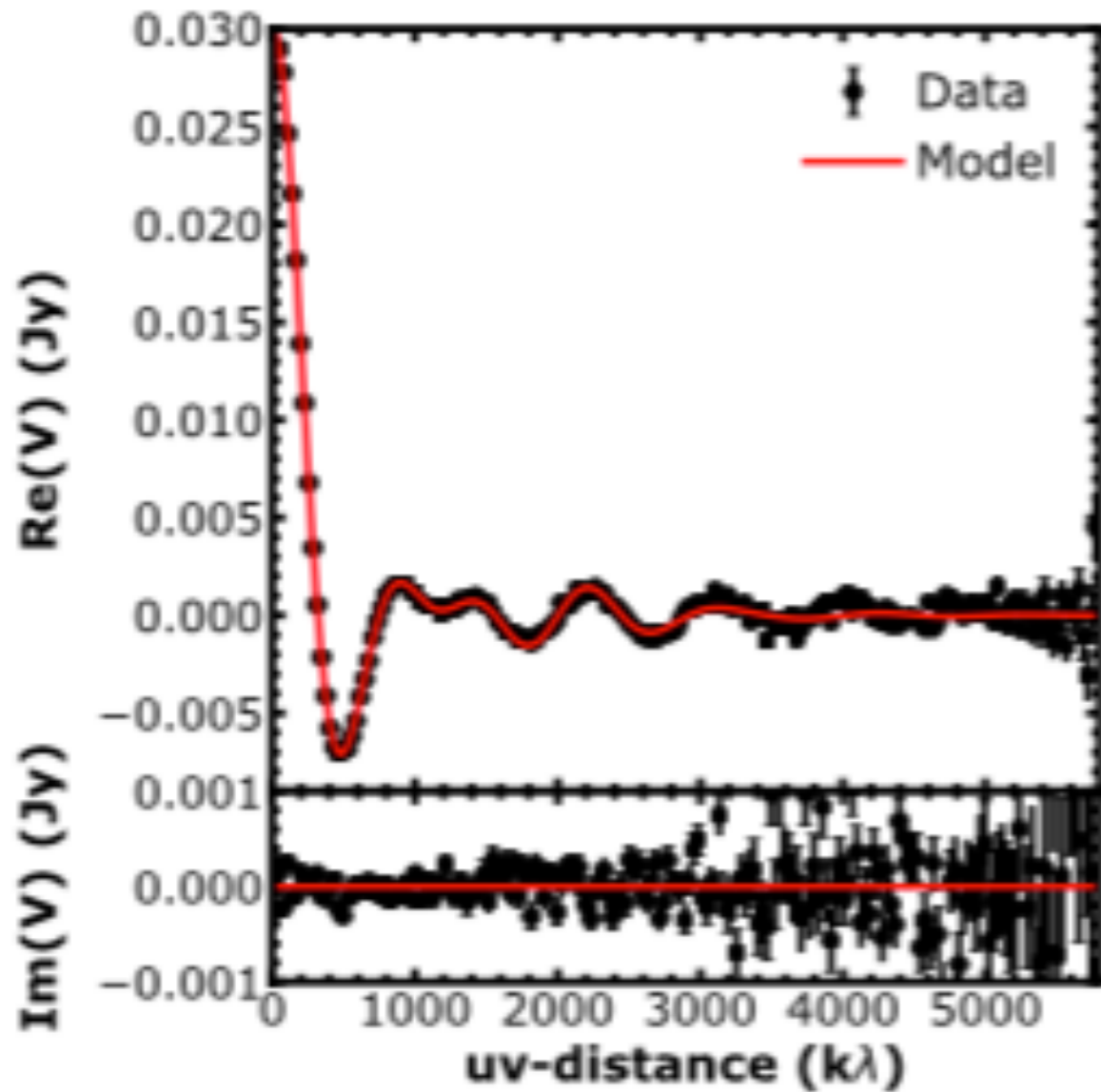
# Interpreting and modeling

## Three flavors:

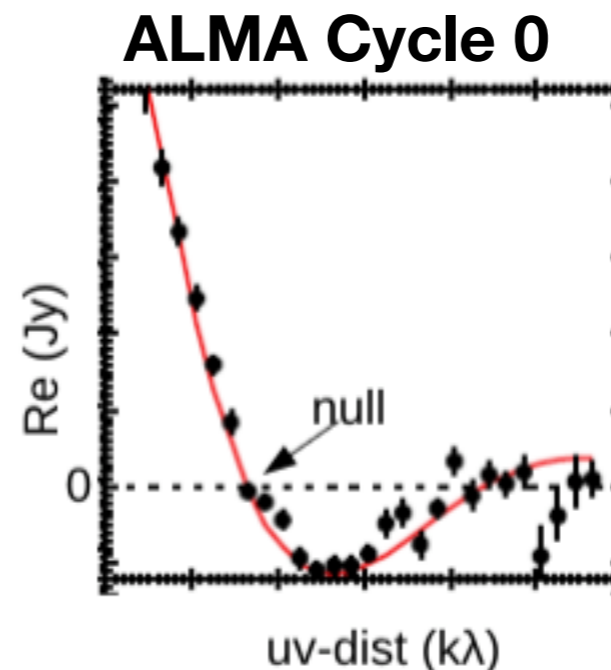
- Full physical-chemical model
  - Small chemistry network + physics (freeze-out, photodissociation) + heating/cooling + excitation + radiative transfer
  - e.g. DALI/ProDiMo
- Line ray tracer
  - Excitation + radiative transfer (abundances parametrized, temperature from dust)
  - e.g. RADMC-3D, LIME
- Full chemical model
  - All of the above with advanced network
  - e.g. codes by Walsh, Aikawa, Cleeves, etc.

**Comparison: spectra/channel maps which can be convolved and compared with data**

# Visibility curves disks



- Due to incomplete uv-coverage, fitting of interferometry data often performed in visibility
- Visibilities also give information on 'smoothness' of radial profile, small scale structures (long baselines)
- Remember: Real (radial), Imag (phase/ azimuthal) and  $Amp = \sqrt{Real^2 + Imag^2}$
- For axisymmetric disks,  $Imag = 0$ , so only Real.

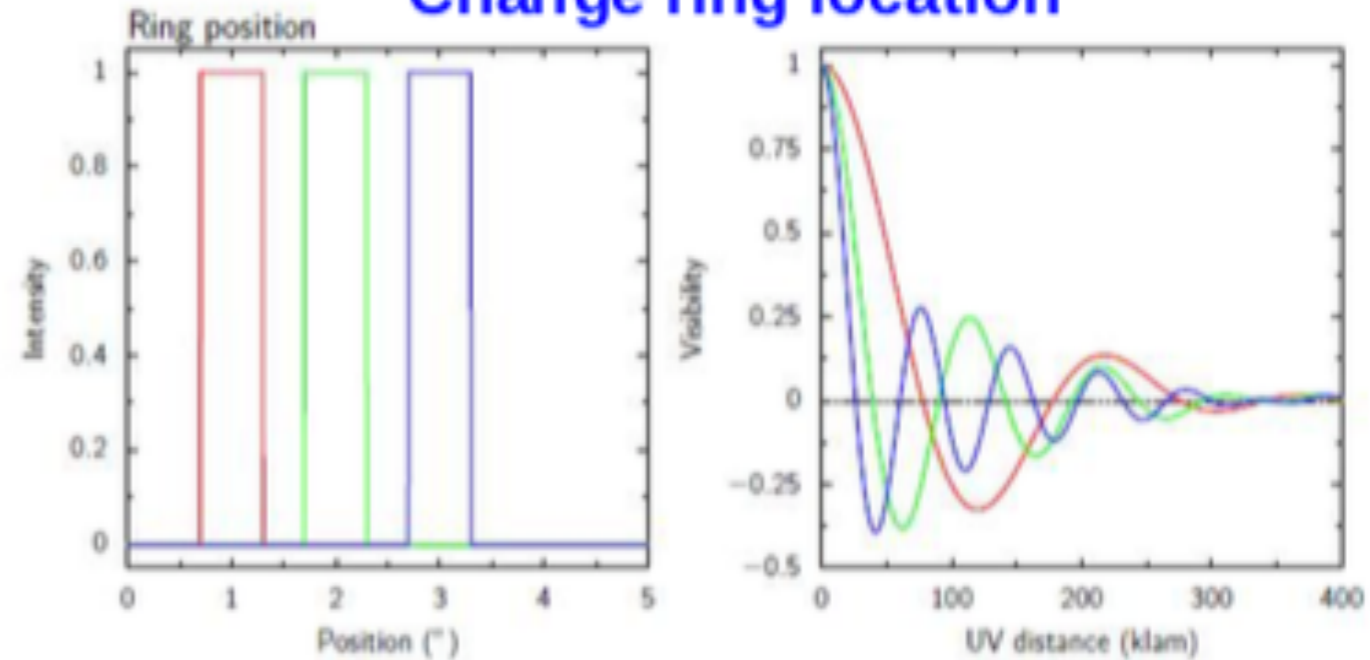


Detection of transition disk cavities used to be limited by lack of long baselines!

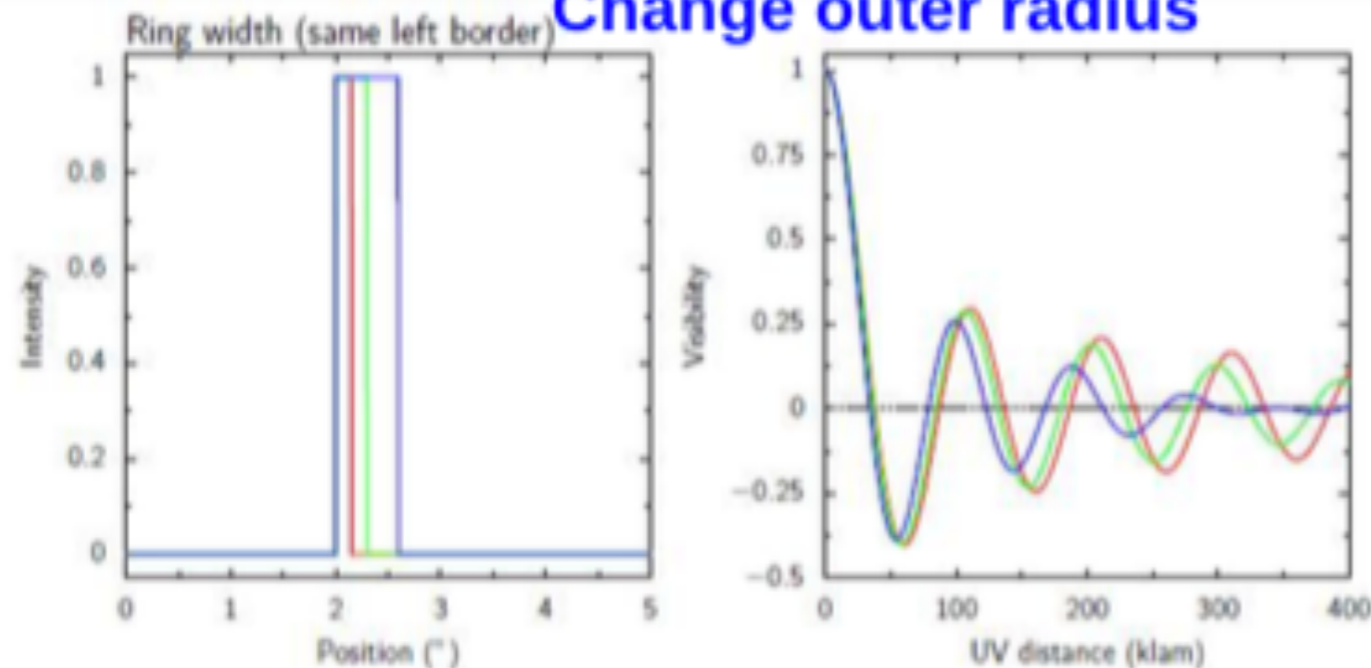
# Visibility curves disks

- Location of the null tells you about the ring location, not (only) the cavity size
- Amplitude of ringing tells you about the ring width

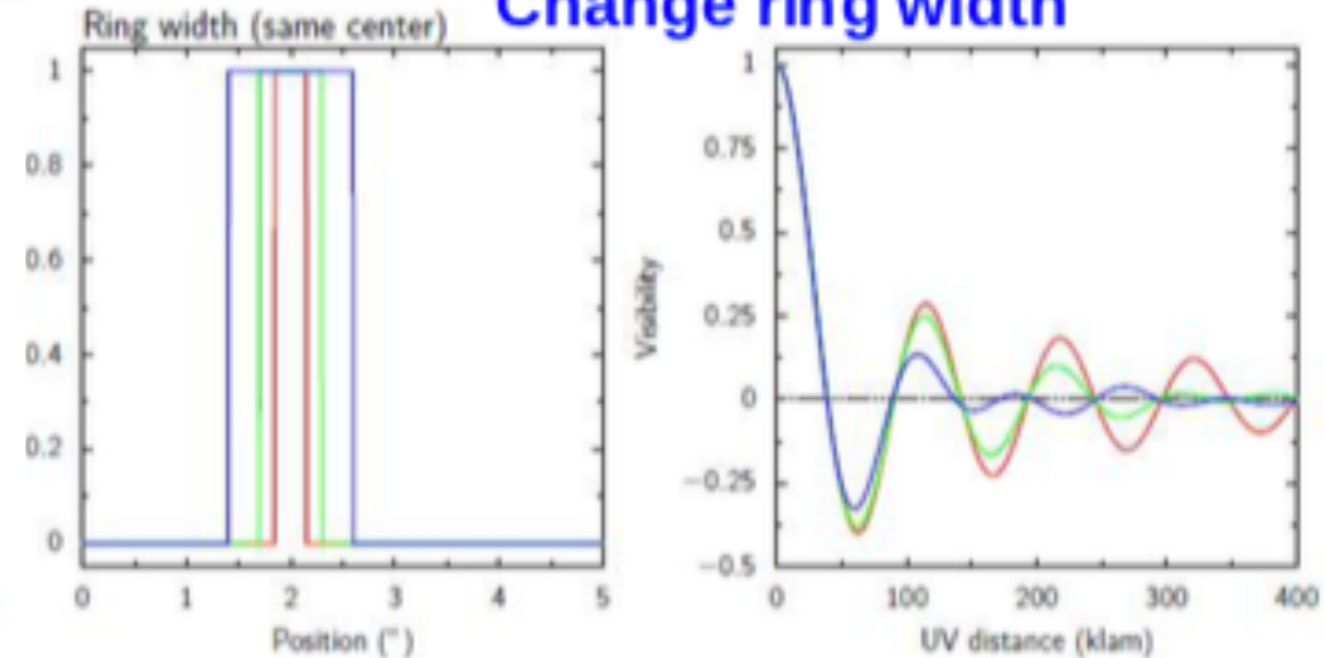
**Change ring location**



**Change outer radius**

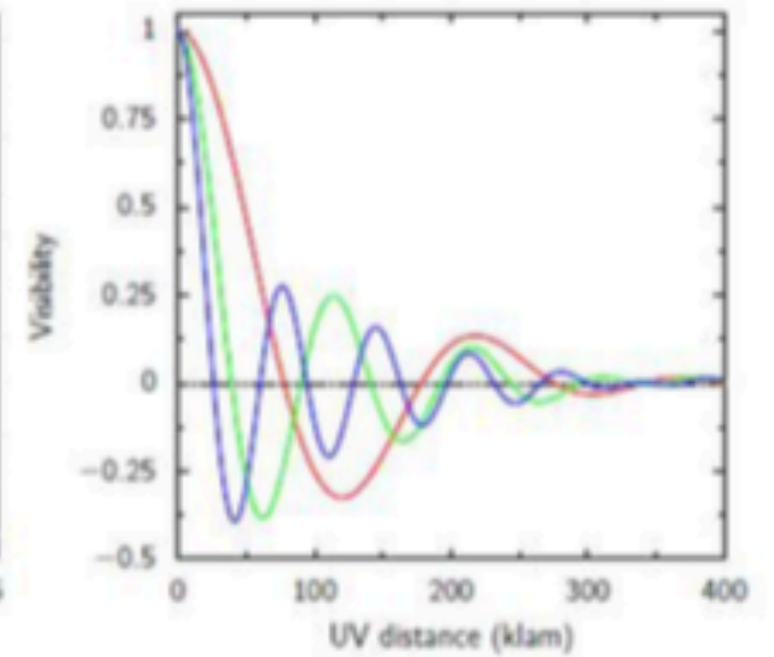
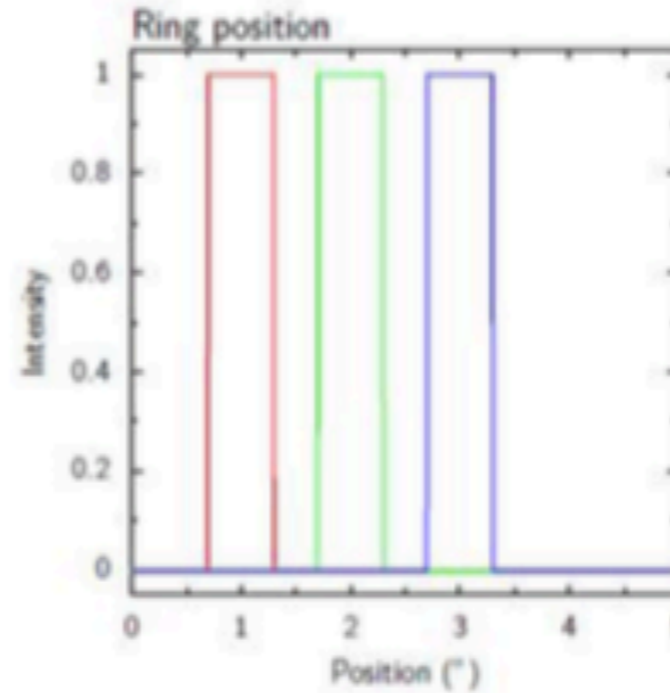


**Change ring width**

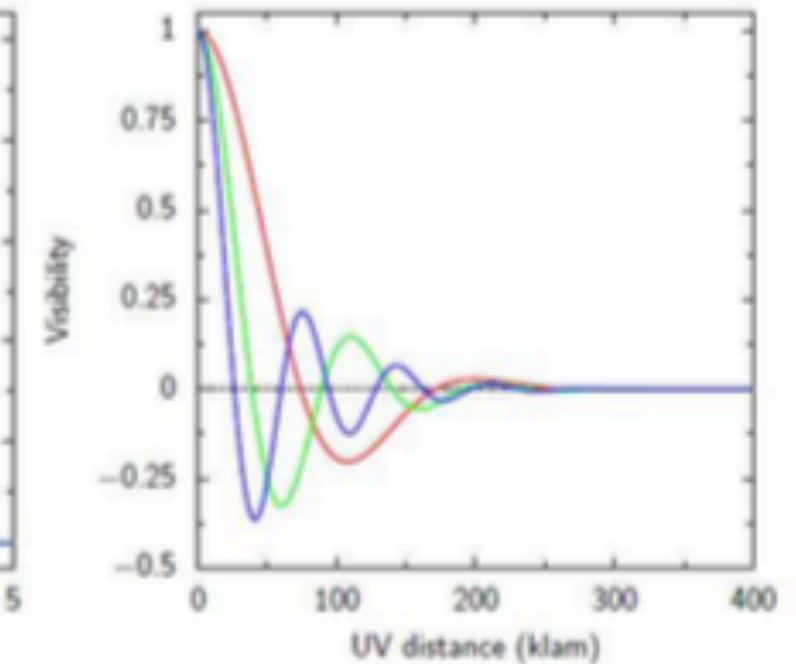
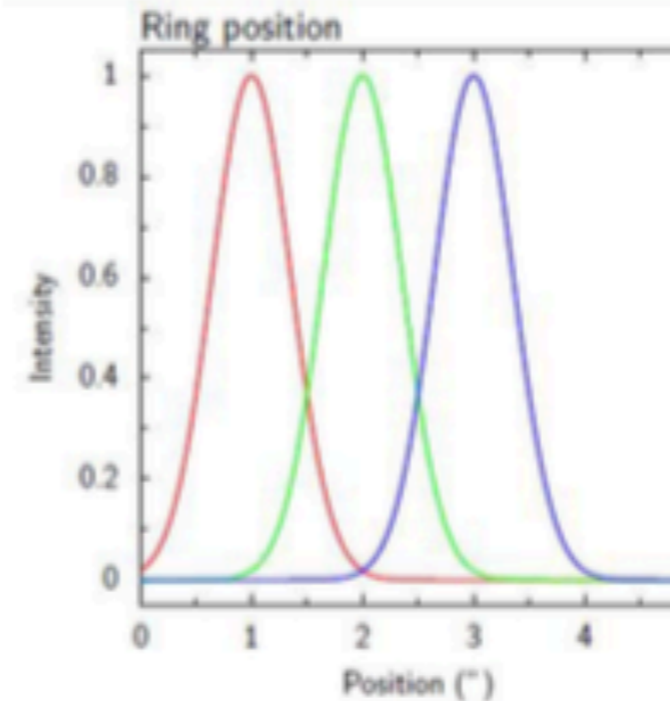


# Visibility curves disks

Sharp



Smooth



- Amount of 'ringing' tells you about the presence of sharp edges

# Visibility curve disks

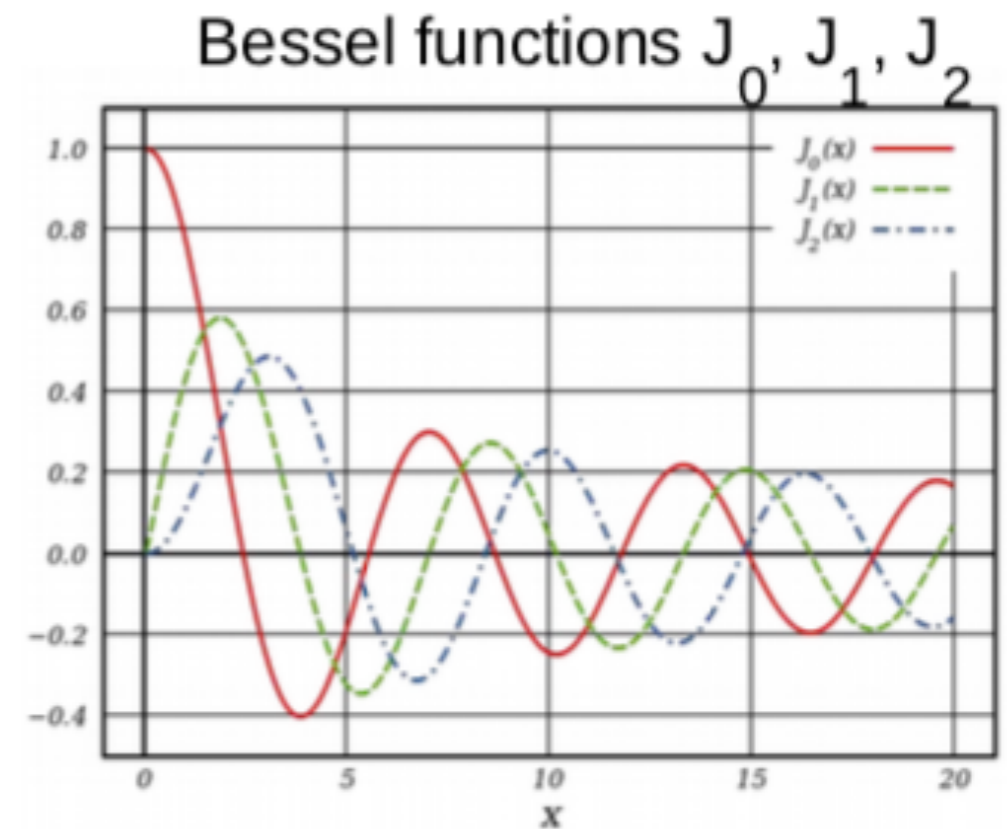
- For a 1D radial profile, one can use an analytical function to compute the Real visibility

$$V_{\text{Re}}(r_{uv}) = 2\pi \int_0^\infty I(\theta) J_0(2\pi r_{uv} \theta) \theta d\theta$$

With Bessel function  $J_0$ :

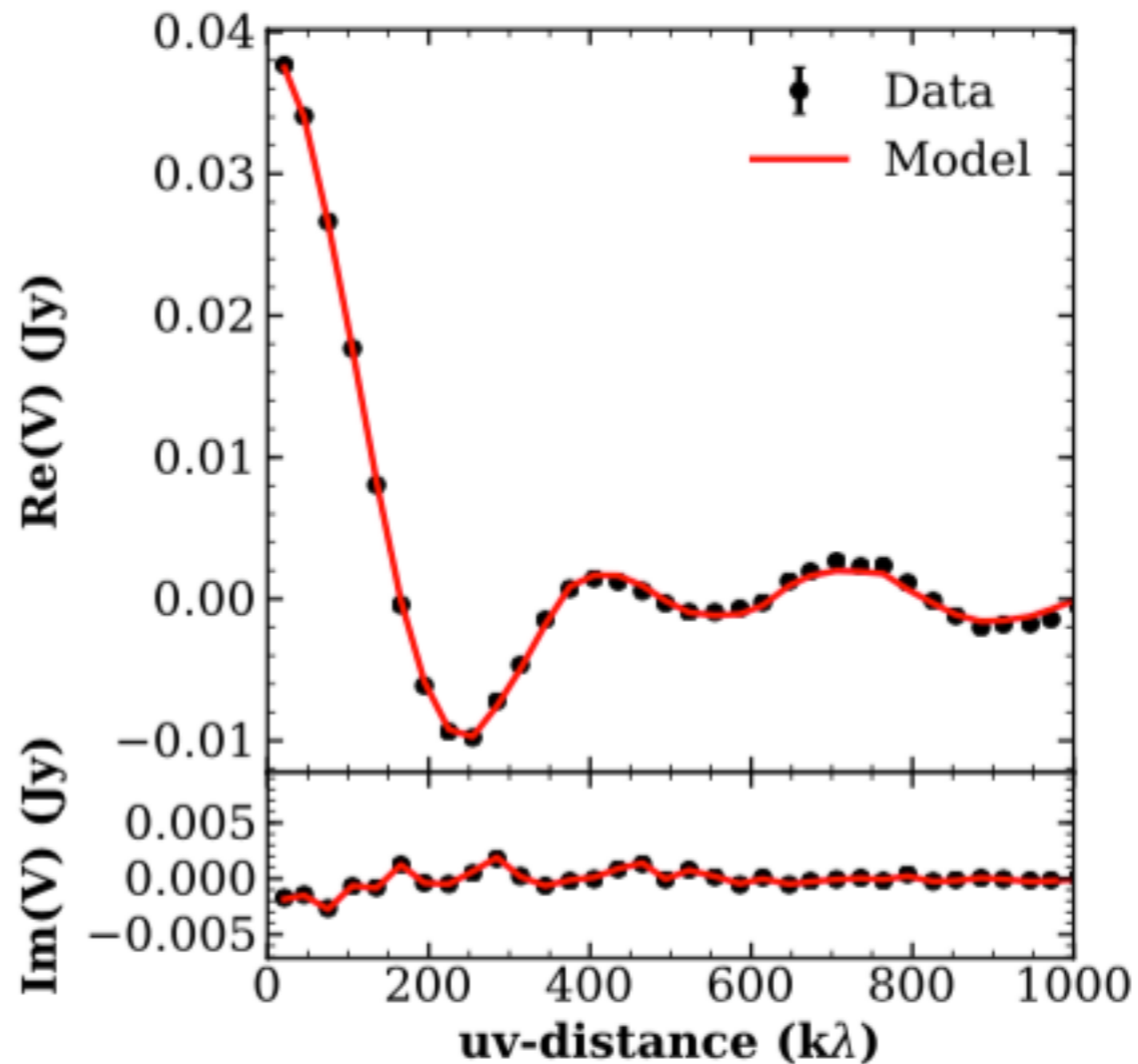
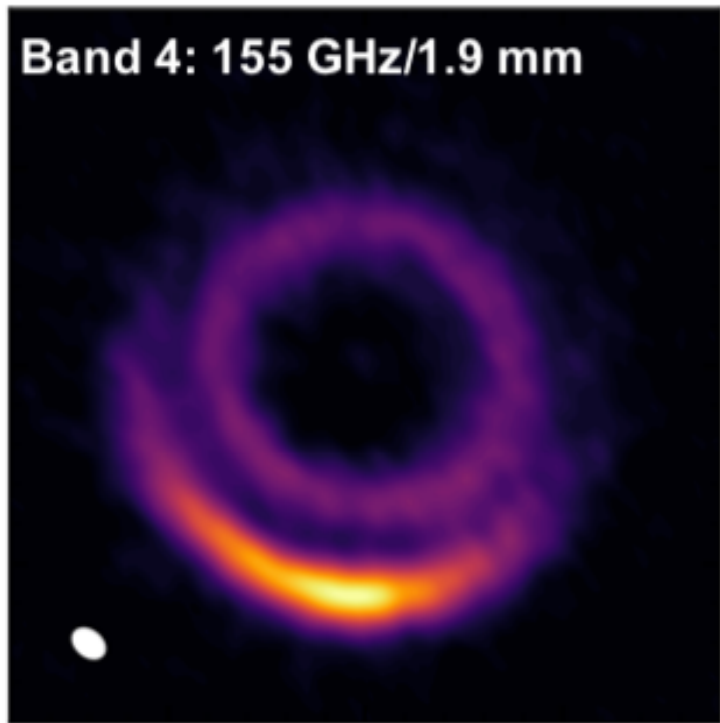
$$J_0(x) = \frac{1}{2\pi} \int_0^{2\pi} \exp(-ix \cos \Theta) d\Theta$$

In practice, use Galaro (Logan)



# Visibility curve disks

- Things more complicated for asymmetries:  
imaginary is non-zero and 2D model required (Galarico)



Model:  $F_1(r, \theta) * F_2(r, \theta)$  with

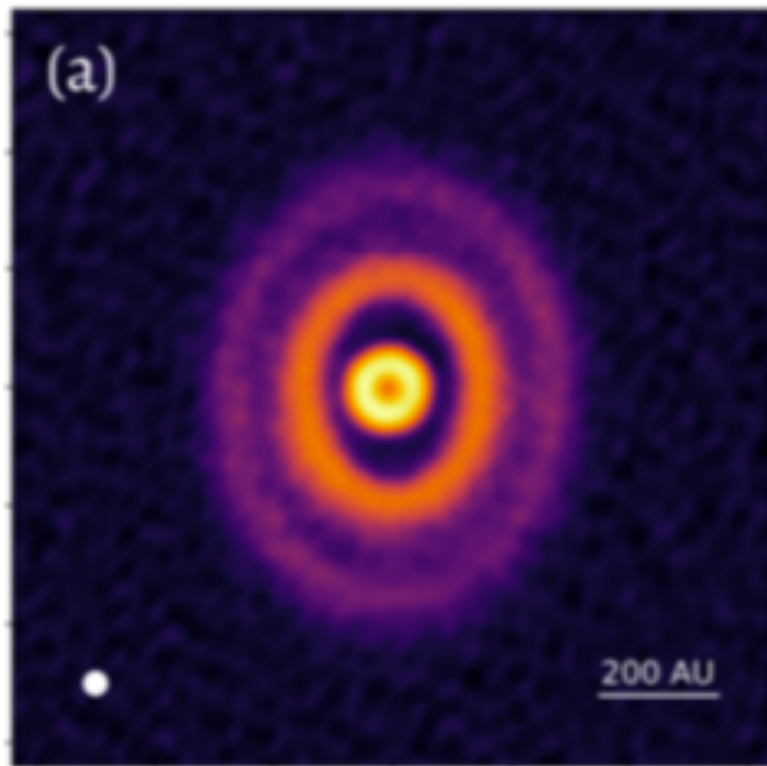
$$F_1(r, \theta) = F_R e^{-(r-r_R)^2/2\sigma_R^2}$$

$$F_2(r, \theta) = \begin{cases} F_V e^{-(r-r_V)^2/2\sigma_{V,r}^2} e^{-(\theta-\theta_V)^2/2\sigma_{V,\theta 1}^2}, & \theta \leq \theta_V \\ F_V e^{-(r-r_V)^2/2\sigma_{V,r}^2} e^{-(\theta-\theta_V)^2/2\sigma_{V,\theta 2}^2}, & \theta > \theta_V \end{cases}$$

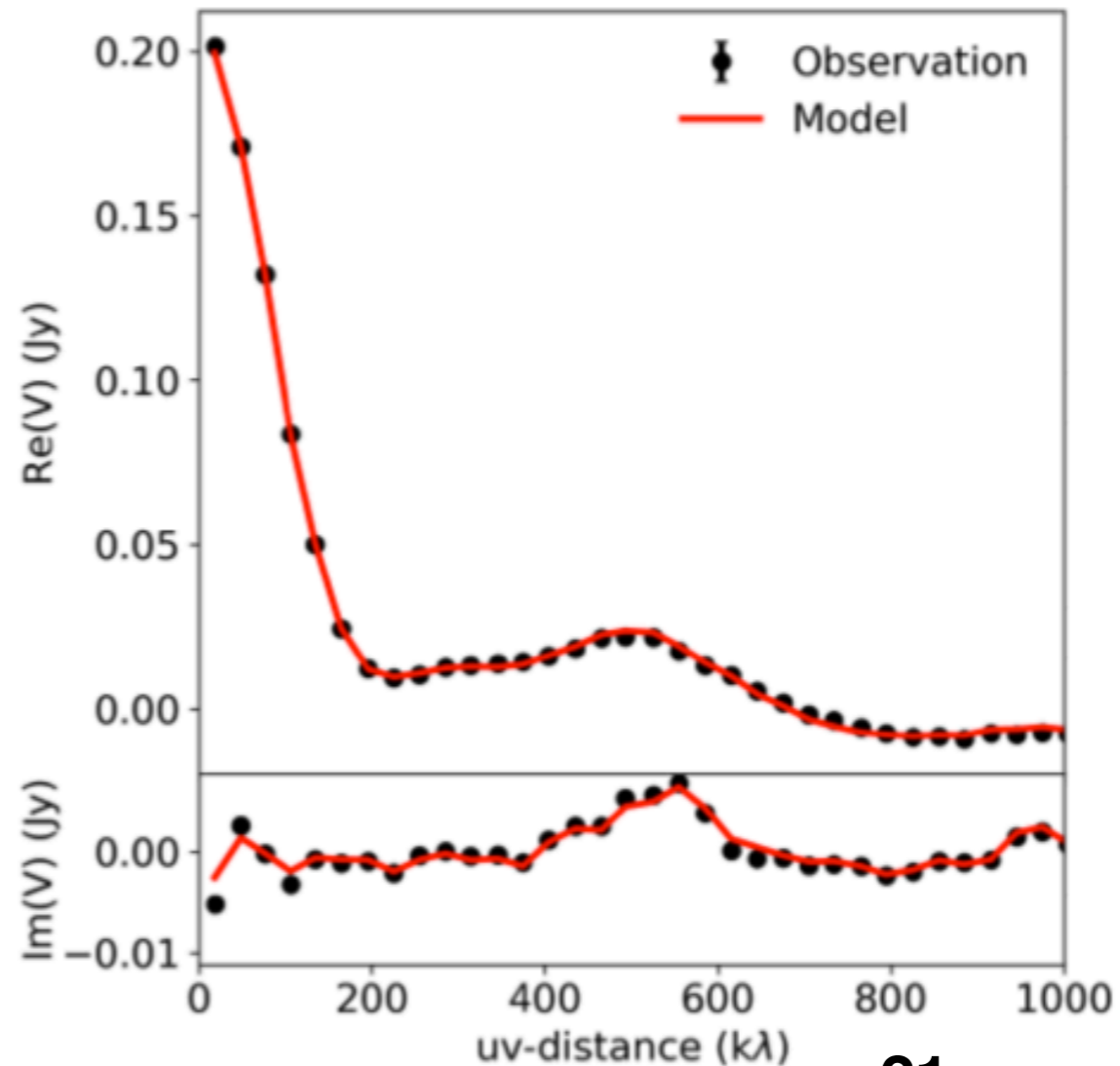
**12 parameters!**

# Visibility curve disks

- And you can go to much more exotic 2D profiles when you have a more complex disk image...



Three rings with different orientations and centers



21 parameters!

# Visibility analysis disks

- Extract a table with u,v,Real,Imag from measurement set using uvplot package

```
B9_uvtable.txt
# Extracted from B9_cont_noflags.ms/.
# wavelength[m] = 0.000449727125448
# Columns:      u[m]      v[m]      Re(V) [Jy]      Im(V) [Jy]      weight
2.219178e+02    -1.450061e+02    4.842058e-01    -3.765531e-01    1.921555e+04
2.757817e+02    -4.283640e+02    2.216566e-02    -2.441918e-01    1.766988e+04
9.776034e+01    -3.275504e+02    -1.399058e-01    2.041570e-02    1.701038e+04
-7.866645e+01   -1.597811e+02    -1.690933e+00    -3.685441e-01    1.751481e+04
6.459532e+01   -1.154262e+02    -9.569053e-02    5.794569e-01    1.915470e+04
-5.563606e+01  -7.828133e+01    1.197055e+00    3.002489e-01    1.766181e+04
3.871151e+01   -1.950632e+02    -3.202173e-01    -1.028351e-01    1.792735e+04
-1.615198e+02  -1.377477e+02    9.605574e-02    -3.192452e-03    1.690295e+04
3.612773e+01   -1.106117e+02    -6.984636e-01    5.463382e-01    2.016426e+04
-7.833507e+01  -4.523733e+02    6.304450e-01    5.525262e-01    1.476806e+04
1.499962e+02   -8.304025e+01    -3.089664e-02    1.733548e-01    1.768173e+04
2.871406e+01   -2.646469e+02    -3.486953e-01    -5.746815e-01    1.688785e+04
4.642532e+00   -4.370372e+02    2.699271e-01    -9.895394e-02    1.981141e+04
-3.288557e+00  -3.156059e+02    4.229355e-01    -1.215404e-01    1.665717e+04
3.837015e+01   -2.477548e+02    6.278983e-02    -1.307128e-01    1.783246e+04
-9.907910e+01  -3.224031e+02    5.438327e-01    4.785762e-01    1.946480e+04
-1.341418e+02  -1.829466e+01    -6.221593e-01    -4.616356e-01    1.882445e+04
2.686930e+02   -2.612326e+02    2.815820e-01    4.510436e-01    1.827388e+04
```

- Deproject the data, compute sqrt and bin for pretty plot

# Visibility analysis disks

- Deprojection means correcting for PA and inclination
- Convention: deproject uv-distances before plotting for consistency between disks

- Equations: 
$$\begin{cases} u_{\Theta} = u \cos \Theta + v \sin \Theta \\ v_{\Theta} = -u \sin \Theta + v \cos \Theta \end{cases}$$

$$r_{uv\Theta i} = \sqrt{u_{\Theta}^2 + v_{\Theta}^2 \cos(i)^2}$$

(if you ever work with SMA: opposite sign direction)

# Visibility analysis disks

- Careful: Real and Imag are extracted from the data w.r.t. phase centre (should be center of disk ring)
- If phase centre is set incorrectly, Imag can become non-zero for axisymmetric disk => incorrectly interpreted as asymmetry
- Phase centre ms-file can set using CASA fixvis()

